

Flavor-dependent eigenvolume interactions in hadron resonance gas and its implications for hadron yields at LHC energies

P. Alba,¹ V. Vovchenko,^{1,2,3} M. I. Gorenstein,^{4,1} and H. Stoecker^{1,2,5}

¹Frankfurt Institute for Advanced Studies, Goethe Universität Frankfurt, D-60438 Frankfurt am Main, Germany

²Institut für Theoretische Physik, Goethe Universität Frankfurt, D-60438 Frankfurt am Main, Germany

³Department of Physics, Taras Shevchenko National University of Kiev, 03022 Kiev, Ukraine

⁴Bogolyubov Institute for Theoretical Physics, 03680 Kiev, Ukraine

⁵GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany

We study the eigenvolume effects in the hadron resonance gas (HRG) model on hadron yields at zero chemical potential. Using different mass-volume relations for strange and nonstrange hadrons we observe a remarkable improvement in the quality of the fit of the mean hadron multiplicities measured by the ALICE Collaboration in the central Pb+Pb collisions at the collision energy $\sqrt{s_{NN}} = 2.76$ TeV. The fit within the point-particle HRG yields $\chi^2/N_{\text{dof}} \simeq 27.1/8$ while the corresponding calculation within the eigenvolume HRG with smaller radii for heavier strange hadrons yields a remarkably small $\chi^2/N_{\text{dof}} \simeq 0.42/6$. This effect appears to be rather insensitive to the details in the implementation of the HRG model, including the variations in the hadron list, as well as the variations in the excluded-volume mechanism. Our result shows that there are no anomalies in the description of the heavy-ion hadron yield data at LHC within the HRG model, as long as physical extensions to the point-particle approximation are permitted. Our finding of the smaller proper volumes for heavy strange hadrons can be also checked by a comparison with the lattice QCD results.

Statistical model approach is an important tool to extract the properties of matter created in relativistic nucleus-nucleus (A+A) collisions (see, e.g., Refs. [1–12]). The experimental hadron multiplicities in A+A collisions have been rather successfully fitted in a wide range of collision energies in terms of a few basic parameters: temperature T , baryonic chemical potential μ_B , and volume V . The most popular version of the statistical model is the *ideal* (point-particle) hadron resonance gas (I-HRG) model. It is based on the idea that the resonance formation mediates the attractive interactions among hadrons. This corresponds to a statistical system of noninteracting hadrons and resonances and leads to the following formula for the system pressure in the grand canonical ensemble

$$p(T, \mu_B) = \sum_j p_j^{\text{id}}(T, \mu_j), \quad (1)$$

where p_j^{id} is the ideal gas pressures, and the sum runs over all known hadron and resonances. The chemical potentials μ_j for j th particle species are taken as

$$\mu_j = b_j \mu_B + s_j \mu_S + q_j \mu_Q, \quad (2)$$

where b_j , s_j , and q_j correspond, respectively, to the baryonic number, strangeness, and electric charge of j th particle. The pressure function (1) depends on two independent variables: temperature T and baryon chemical potential μ_B . The strange chemical potential $\mu_S = \mu_S(T, \mu_B)$ and the electric chemical potential $\mu_Q = \mu_Q(T, \mu_B)$ are found from the conditions of the zero net strangeness and the fixed electric-to-baryon charge ratio in the colliding nuclei. The other intensive thermodynamical functions (e.g., particle number densities, energy density, and entropy density) are obtained from (1) by standard thermodynamic formulae.

In order to account for the short-range repulsive interactions between hadrons a thermodynamically consistent *excluded volume* (EV) van der Waals procedure was suggested [13]. For the multicomponent HRG the simplest formulation of the EV model leads to a transcendental equation [5]

$$p(T, \mu_B) = \sum_j p_j^{\text{id}}(T, \mu_j^*), \quad (3)$$

$$\mu_j^* = \mu_j - v_j p(T, \mu_B). \quad (4)$$

with $v_j = 16\pi r_j^3/3$ being the eigenvolume parameter for the particle j , and r_j corresponds to its effective hard-core radius¹. The particle number densities of i th species are then calculated as

$$n_i(T, \mu_B) = \frac{n_i^{\text{id}}(T, \mu_i^*)}{1 + \sum_j v_j n_j^{\text{id}}(T, \mu_j^*)}. \quad (5)$$

The I-HRG model is usually considered as a baseline with regards to both the fit of the data on mean hadron multiplicities and the comparison with the lattice QCD results at $\mu_B = 0$ (see, e.g., Ref. [15–17]). We think however that repulsive interactions between hadrons due to non-zero size of particles are both physically justified and important (see, e.g., Refs. [18, 19]). Some particular implementations of the EV-HRG model have been confronted to the lattice QCD data [18, 20, 21]. In the present letter we discuss the fit of mean hadron multiplicities and possible role of the EV effects. How these

¹ The relation $v_j = 16\pi r_j^3/3$ is valid only in the case when the influence of the quantum-mechanical effects on a hard-core interaction is negligible. In general the classical formula may notably underestimate the 2nd virial coefficient (see e.g. Ref. [14]) and one should rather treat r_j as an *effective* hard-core radius.

particular effects influence the comparison with the lattice QCD data will be considered elsewhere.

Equation (5) gives the *primordial* equilibrium density of stable hadrons and resonances in A+A collisions. Their total numbers N_i are obtained by multiplying the n_i by the system volume, $N_i = V n_i$. The mean *final* multiplicity $\langle N_h \rangle$ is calculated in the HRG model as a sum of the primordial mean multiplicity and resonance decay contributions as follows

$$\langle N_h \rangle = V n_h + V \sum_R \langle n_h \rangle_R n_R, \quad (6)$$

where $\langle n_h \rangle_R$ is the average number of particles of type h resulting from a decay of resonance R .

As seen from Eq. (5) the EV procedure introduces a suppression factor $[1 + \sum_j v_j n_j^{id}(T, \mu_j^*)]^{-1} < 1$, which is the same for all type of particles, and also an additional suppression due to the shift of chemical potential as given by Eq. (4). For classical (Boltzmann) approximation this leads to an additional factor $\exp[-v_i p(T, \mu_B)] < 1$. In the case of $v_i = \text{const}$ for all particle species the overall suppression of each hadron density n_i due to the EV effects compared to their ideal gas values n_i^{id} is essentially the same, with small differences resulting due to the quantum-statistical effects. Therefore, the particle number ratios are almost unchanged, and rescaling the total volume V one may then obtain the $\langle N_h \rangle$ values (6) equal to those in the I-HRG. As a consequence, the $v_i = \text{const}$ case yields essentially no changes in the behavior of the thermal fits to hadron yield data. This fact has led to a relatively common misconception that the inclusion of the eigenvolume interactions into HRG has a negligible effect on the thermal fits. However, if v_i are chosen to be different for different i , the suppression will be stronger for particles with a larger eigenvolume. In such a case the description of the hadron ratios is affected notably (see e.g. Refs. [5, 22]), and in some cases the effect can be quite dramatic [23].

We employ the EV-HRG model (3-5) to describe the experimental data on particle yields for π^+ , π^- , K^+ , K^- , p , \bar{p} , Λ , Ξ^+ , Ξ^- , and $\Omega + \bar{\Omega}$ measured in 0-5% central Pb+Pb collisions at the center of mass energy of nucleon pair $\sqrt{s_{NN}}=2.76$ TeV by the ALICE Collaboration at the Large Hadron Collider (LHC) of European Organization for Nuclear Research (CERN) [24–26]. The experimental centrality binning for Ξ and Ω hyperons is different from the other hadrons. Thus, we take the midrapidity yields of Ξ and Ω in the 0–5% centrality class from Ref. [27], where they were obtained using the interpolation procedure. Our own implementation of the HRG model is used in the analysis. In our calculations we include hadrons and resonances that are listed by the Particle Data Group [28] and take into account the quantum statistics (see [29–31] for the details of the implementation). Note that we do not include the light nuclei, neither in the fit nor in the particle list.

We have checked that the deviations of the baryonic chemical potential μ_B from zero are negligible, this im-

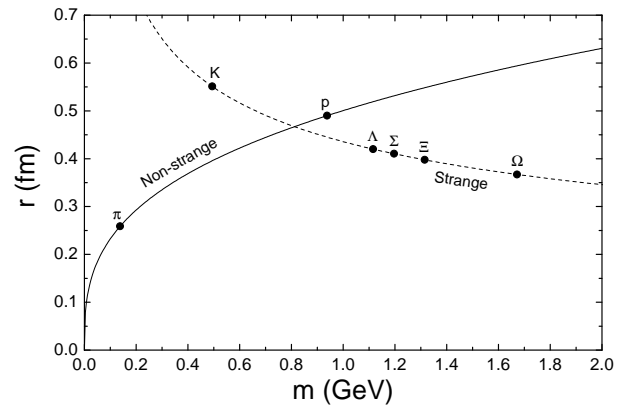


FIG. 1. The effective hard-core radii of nonstrange (solid line) and strange (dashed line) hadrons given by Eqs. (8) and (9), respectively. The constants α and γ are fixed in order to reproduce the proton radius of $r_p = 0.49$ fm and the Λ radius of $r_\Lambda = 0.42$ fm.

mediately leads also to $\mu_S = \mu_Q = 0$. Therefore, the free parameters in the model are the temperature T and the system volume per unit of rapidity V in the I-HRG, and, additionally, one or several parameters which regulate the hadron eigenvolumes in the EV-HRG. The HRG model fits are done by minimizing the value of

$$\frac{\chi^2}{N_{\text{dof}}} = \frac{1}{N_{\text{dof}}} \sum_{h=1}^N \frac{(\langle N_h^{\text{exp}} \rangle - \langle N_h \rangle)^2}{\sigma_h^2}, \quad (7)$$

where $\langle N_h^{\text{exp}} \rangle$ and $\langle N_h \rangle$ are the experimental and theoretical hadron yields, respectively, N_{dof} is the number of degrees of freedom, that is the number of the data points minus the number of fitting parameters, and σ_h are the experimental errors on hadron yields.

It was recently pointed out that a bag-like parameterization for the eigenvolume would improve the description of yields at the LHC [23], as well as at the lower collision energies [32]. This implies the linear mass-volume relation (m_i is a mass of i th particle)

$$v_i = \alpha m_i, \quad (8)$$

with the same parameter α for all particle species. In the present study we explore the specific dependence for the particle eigenvolumes. It is assumed that Eq. (8) is valid for all hadrons with zero strangeness, both mesons and (anti)baryons. On the other hand, for the strange particles, the following *inverse* mass-volume relation is employed:

$$v_s = \gamma m_s^{-1}. \quad (9)$$

This can be justified phenomenologically by the smaller cross sections for the heavier strange hadrons, and this can affect the description of the strange hadrons within a thermal fit [33]. Note that parameters α and γ can

be fixed by specifying the proton radius r_p and the Λ radius r_Λ in Eqs. (8) and (9), respectively. Our fit to the data reveals that the best description of the mean multiplicities at LHC is given by the bag-like relation (8) for non-strange hadrons with $r_p = 0.49$ fm and inverse-bag parameterization (9) for strange particles with $r_\Lambda = 0.42$ fm. The resulting radii for different hadrons are shown in Fig. 1. The calculation results and comparison to the I-HRG are presented in Table I. The fit within the I-HRG yields a $\chi^2 \simeq 27.1$ with chemical freeze-out temperature of $T \simeq 153$ MeV. Our result appears to be generally consistent with previous fits performed by different groups [27, 34–36]. The fitting procedure for the EV-HRG with Eqs. (8) and (9) leads to an anomalously small value of $\chi^2 \simeq 0.42$ at the global minimum. We have also checked that employing the bag model parametrization Eq. (8) with parameters for both strange and non-strange hadrons yields at best a χ^2 of about 15, while using the inverse mass-volume relation (9) for both gives always results worse than the I-HRG. We further checked the possibility for constant radii for strange and non-strange mesons and baryons, adding 4 parameters in total; in general we were not able to obtain a critical improvement of the particle fit, but this simple assumption confirms that strange hadrons need for a smaller EV suppression; similar results were obtained by [37–40] for lower collision energies, where however a much larger number of parameters has been introduced. This will definitely require additional studies and discussions.

	χ^2/N_{dof}	T (MeV)	V (fm ³)
I-HRG	$27.1/8 \cong 3.39$	153.0 ± 2.3	5440
EV-HRG	$0.42/6 \cong 0.07$	151.4 ± 1.8	11008

TABLE I. Temperature T and volume V which minimize the χ^2 of the fit to the ALICE data on mean multiplicities in 0–5% most central Pb+Pb collisions at $\sqrt{s_{\text{NN}}} = 2.76$ TeV in the I-HRG, and in the EV-HRG with $\alpha = 2.10$ fm³/GeV ($r_p = 0.49$ fm) in Eq. (8) and $\gamma = 1.38$ fm³·GeV ($r_\Lambda = 0.42$ fm) in Eq. (9), respectively.

In Fig. 2 the temperature dependence of the χ^2 within the I-HRG, and the EV-HRG with $r_p = 0.49$ fm and $r_\Lambda = 0.42$ fm, is presented. At each value of the temperature the total system volume per rapidity is fixed in order to minimize the χ^2 . The width of the χ^2 in both considered cases is similar. It is worth to note that a second minimum structure appears for very high temperatures as one could expect from ground arguments [23], which however we neglect it here because it is 3 orders of magnitude greater than the true minimum.

In Fig. 3 we show the ALICE yields in comparison with the model calculations. It is clearly seen that the mass dependent eigenvolume drastically reduces all the deviations from central values for all particles species and for all yields the deviations are significantly below the one-sigma deviation. The general improvement of the fit

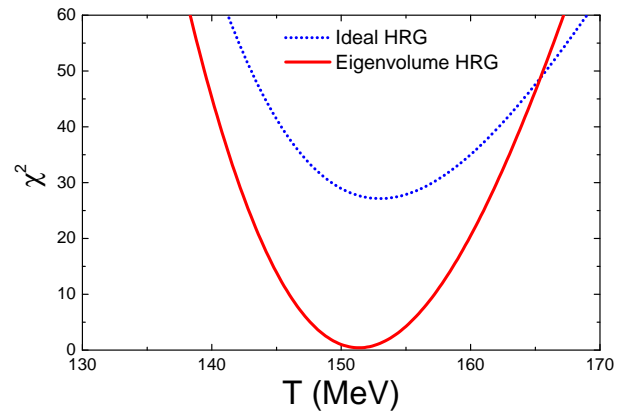


FIG. 2. Temperature dependence resulting from the ALICE data best fit procedure for χ^2 . A dashed line corresponds to the I-HRG and a solid one to the EV-HRG.

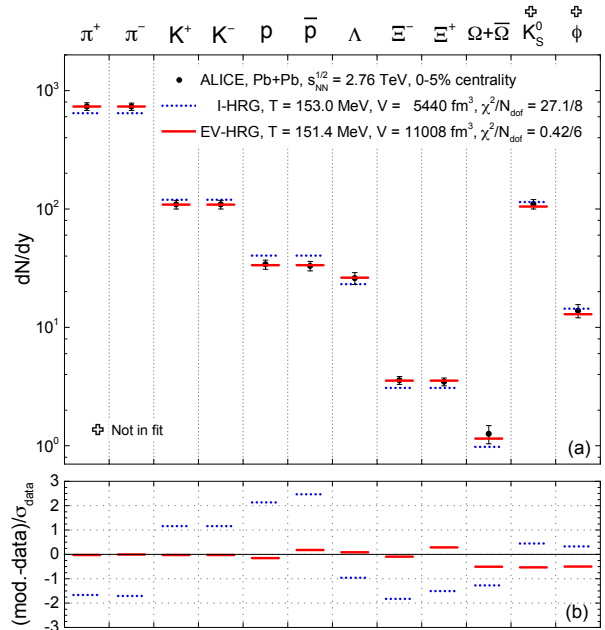


FIG. 3. ALICE yields in comparison with the HRG model calculations. Also shown are the yields of K_S^0 and ϕ mesons which were not originally included in the fit.

can be understood in terms of a balanced suppression for kaons and protons, together with an enhancement for the other hadrons due to an increase of the system volume. The yields of K_S^0 and ϕ , which were not included in the fit, are also shown and it is seen that they are also described well, with deviations not exceeding the half-sigma level. We checked that the inclusion of K_S^0 and ϕ in the fit procedure does not noticeably alter our results. The remarkably small value of $\chi^2/N_{\text{dof}} \simeq 0.42/6 \cong 0.07$ obtained within the EV-HRG model may indicate that experimental uncertainties are overestimated.

It is seen from Fig. 3 that there are no anomalies in

the description of the hadron yield data in Pb+Pb collisions at LHC within the EV-HRG model. In particular the “non-thermal” p/π ratio, which is somewhat poorly described by the I-HRG and which has sparked some discussion [35, 36, 41, 42], is described very well solely due to the eigenvolume interactions. According to our preliminary calculations, a similarly remarkable improvement of the ALICE yield data description within the EV-HRG with the same eigenvolume parameters also takes place for all other ALICE centrality windows.

In order to check the sensitivity of the obtained results with regards to the details of the HRG model implementation, we have also performed the calculations within the different HRG code which was used and described in more detail, e.g., in Refs. [12, 23, 32]. In particular, this implementation has a somewhat different particle list, includes finite width of the resonances, and includes two different implementations of the EV-HRG for a multicomponent system: the “Diagonal” EV model which is given by Eqs. (3-5) and which was used throughout the present work, and the “Crossterms” EV model which was originally formulated in Ref. [43] and which treats correlations between hadrons of different size in a more elaborate way. The fit within I-HRG with this implementation yields $\chi^2 \simeq 30.0$ with $T \simeq 153.3$ MeV and $V \simeq 5440$ fm³. For the EV-HRG with the eigenvolume parametrization shown in Fig. 1, i.e., for $r_p = 0.49$ fm and $r_\Lambda = 0.42$ fm, we obtain the following: the fit within the “Diagonal” EV yields $\chi^2 \simeq 2.07$ with $T \simeq 151.5$ MeV and $V \simeq 11124$ fm³ while the fit within the “Crossterms” EV gives $\chi^2 \simeq 2.51$ with $T \simeq 152.5$ MeV and $V \simeq 10302$ fm³. It is likely that χ^2 can be made even smaller by varying the r_p and r_Λ . We conclude that a remarkable improvement of the fit quality upon introduction of the smaller eigenvolumes for the heavier strange hadrons is quite robust with regards to the details of the modeling of the eigenvolume effect, as well as to the used particle list and the inclusion of the finite widths of the resonances.

It is also interesting to apply the EV-HRG model (3) and (4) for A+A collisions at smaller energies. The finite baryon chemical potential will play an additional

role there. Our preliminary calculations show that the the considered eigenvolume parametrizations (8) and (9) with the same parameters α and γ give a systematic improvement in the description of the hadron yield data at energies of the Super Proton Synchrotron (SPS) at CERN. It may also be worthwhile to extend this study to the fluctuations of the conserved charges, which may be more sensitive to the finer details of the QCD phase diagram [44].

In summary, we explore the EV effects in the HRG model with a flavor dependent mass-volume relation: the bag model mass-proportional relation for non-strange hadrons and the inverse mass-volume relation for the strange ones. Using these mass-volume relations we observe a remarkable improvement in the fitting of the mean hadron multiplicities measured in central Pb+Pb collisions at the CERN LHC by the ALICE Collaboration. These results indicate that there are no anomalies in the description of the available hadron yield data in Pb+Pb collisions at LHC within the HRG model, as long as physical extensions to the point-particle approximation are permitted. It also implies that the simple point-particle HRG model has a rather limited applicability and it should be used with care. In particular one should not draw too many physical conclusions from analysis performed within the point-particle HRG. A systematic comparison of the EV-HRG model employing the presented mass-volume relations with the lattice QCD, as well as the analysis of the hadron yield data at lower collision energies, will be explored further.

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